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Keywords: Public transport, Travel time variability, Reliability, Travel time distribution, Probability, Indicators

Public Transport Travel-Time Variability Definitions and Monitoring

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Abstract

Public Transport Travel Time Variability (PTTV) is essential for understanding the deteriorations in the reliability of travel time, optimizing transit schedules and route choices. This paper establishes the key definitions of PTTV in which firstly include all buses, and secondly include only a single service from a bus route. The paper then analyzes the day-to-day distribution of public transport travel time by using Transit Signal Priority data. A comprehensive approach, using both parametric bootstrapping Kolmogorov-Smirnov test and Bayesian Information Creation technique is developed, recommends Lognormal distribution as the best descriptor of bus travel time on urban corridors. The probability density function of Lognormal distribution is finally used for calculating probability indicators of PTTV. The findings of this study are useful for both traffic managers and statisticians for planning and analyzing the transit systems.

Keywords: Public transport, travel time variability, reliability, travel time distribution, probability, indicators

INTRODUCTION

Public Transport Travel Time Variability (PTTV) is essential for transit operators. It facilitates investigating the deteriorations of travel time reliability and explaining the reliability index. Knowledge of PTTV also simplifies the optimization of recovery time, which is the added time to the schedule running time, to account for both travel time variation and a short break before the next departure. PTTV plays an important role in traveler trip planning and route choice (Abdel-Aty et al. 1995) since unreliable and highly variable travel time increases anxiety, stresses (Bates et al. 2001) and cost to the travelers (Noland and Polak 2002). Therefore, ridership is lost when PTTV is high. A study in Oregon, US found that a 10% decrease in headway delay variation led to an increase of 0.17 passengers per trip per timepoint (Kimpel et al. 2000).

Travel Time Variability (TTV) has been defined in the literature as the variance in travel times of vehicles travelling similar trips (Bates et al. 1987; Noland and Polak 2002). However, the definition is better suited for measuring private rather than public transport, as confusion arises in the definition of "similar trips". While private transport vehicles are treated as homogenous to some extent, public transport vehicles are noticeably different. By stopping at only selected stops, express routes are significantly faster than local routes, questioning the definition of "similar trips" particularly for practical purposes. Conversely, the availability of individual travel time data of each transit vehicle will provide new approaches to better define PTTV.

This paper exploits Transit Signal Priority (TSP) data to establish PTTV's definitions and investigate its statistical characteristics. Firstly, the paper establishes oriented definitions of PTTV, which is based on and also distict from the common definitions used in private transport. Secondly, a comprehensive hybrid approach to investigate the distribution of public transport travel time, considering all types of continuous distribution types is proposed to explore the nature and shape of public transport travel time. Finally, the paper develops a probabilistic indicator of the PTTV, which facilitates the calculation of slack time/recovery time and statistical studies of travel time. These findings enable transport managers and researchers to better plan public transport systems.

TRAVEL TIME VARIABILITY IN LITERATURE

TTV has been defined in the literature as having three main types (Bates et al. 1987; Noland and Polak 2002):

Vehicle-to-vehicle (or inter-vehicle) variability (TTV_{v2v}) is the difference between travel times experienced by different vehicles travelling similar trips within the same time period. Factors contributing to TTV_{v2v} includes signal delay, driver behavior and flow impedance from bikes and pedestrians.

Period-to-period (inter-period or within-day) (TTV_{p2p}) is the variability between the travel times of vehicles travelling similar trips at different times on the same day. Factors contributing to TTV_{p2p} includes temporal variations in traffic demand, incidents, weather conditions or level of daylight.

Day-to-day (or inter-day) (TTV_{d2d}) is the variability between similar trips on different days within the same time period. It is attributed to the day-to-day fluctuations in traffic demand, weather, driver behaviors, and incidents. TTV_{d2d} is independent to the recurrent congestion effects. Within the same time period, a high demand transit system has low day-to-day TTV if congestions are recurrent.

This paper focuses only on the day-to-day PTTV as it is the most advisable and practical type of TTV in public transport. For transit passengers, the variability of travel time of the same service or route on multiple days is more important than the TTV_{v2v} or TTV_{p2p}. Daily commuters travel by a specific route/service at around a specific time of the day (Kieu et al. 2014). For transit operators, day-to-day TTV provides a complete picture of transit performance on multiple days; facilitates schedule optimization and identify the sources of travel time unreliability.

The literature on day-to-day PTTV is limited. Abkowitz and Engelstein (1983) predicted the running time and running time deviation by using linear regression. Their model revealed that only the link length has significant impact on the day-to-day PTTV. Mazloumi et al. (2010) explored the day-to-day PTTV in Melbourne, Australia using GPS data. The nature and pattern of variability were explored by fitting bus travel time to Normal and Lognormal distribution, followed by a linear regression analysis to investigate the impacts of different factors to PTTV. Moghaddam et al. (2011) proposed a procedure and empirical models for predicting the Standard Deviation (SD) of travel time based on the average bus travel time, number of signalized intersection and a ratio between volume and capacity for an origin-destination path.

Abkowitz and Engelstein (1983), Mazloumi et al. (2010) and Moghaddam et al. (2011) have defined PTTV as deviation or SD of travel time using individual bus travel time samples from multiple days at the same time period. Their definitions were based on the common definition of TTV_{d2d} by Bates et al. (1987) and Noland&Polack (2002), where "similar trips" means vehicles of the same route travelling within the same time period.

The estimation of day to day PPTV based on the individual vehicle travel values does not provide true actual daily variations. The calculated PTTV could be sourced from multiple days or multiple vehicles travel time variation because on each day multiple samples are collected. The problem can be explained with the help of an example: Given two days with exactly the same individual vehicle travel times, in which *day1* has *n* buses for a given period. There is TTV_{v2v} during that period if not all travel time values are same. Assuming, *day2* be exactly the same as *day1*, if all the individual vehicle travel time samples from *day1* and *day2* are used to calculate TTV_{d2d} then estimated TTV_{d2d} will be equal to TTV_{v2v}. However, in this example the TTV_{d2d} should be zero because the two days are exactly the same. To the best of the authors' knowledge, there is no paper in the literature established the oriented definitions and explore the statistical characteristic behind PTTV.

DATASET

The TSP sensors are operating at major corridors in Brisbane to give priority to buses at the signalized intersections. The sensors act as an automatic vehicle monitorin system to identify the unique vehicle identification number, route, timestamps and service scheduled start times of each passing bus. The difference between observed timestamps at upstream and downstream intersections is the travel time between the two intersections.

Figure 1 shows 4 major arterial corridors in Brisbane along with their operating bus routes and lengths. The Coronation Drive corridor (from High Street to Cribb Street) is the case study site for PTTV definition establishment and analysis in this paper. The study site is highly congested on both morning and afternoon peak periods. The other three corridors and their bus routes are used at the final sub-section of the analysis for validation.

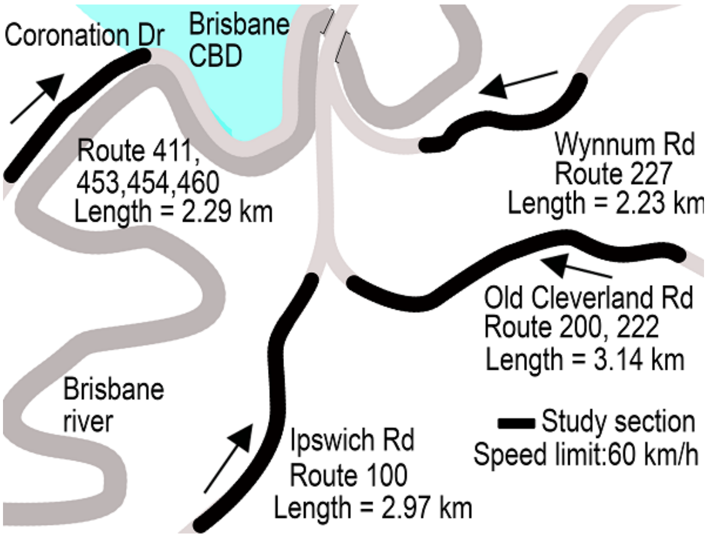


Figure 1 Study site [INSERT]

The analysis has been carried out on a year of TSP data (1st July 2011 to 30th Jun 2012) on inbound traffic. The analysis performed in this paper is on the recurrent variability of travel time of in-service buses (buses that are on operation) during working days (weekdays excluding Public Holidays and School Holidays). Public transport data is integrated with incident records to filter out travel time values during incidents. Service scheduled start time is the scheduled departure time from the depot, which is defined as a "service" in this paper.
Buses started earlier or later than the predetermined scheduled start time are also not considered, since different stop skipping, bus holding or priority strategies could have been applied exclusively on them.

DAY-TO-DAY PUBLIC TRANSPORT TRAVEL TIME VARIABILITY DEFINITIONS

This section establishes two key definitions of PTTV to measure only the day-to-day variation of travel time, considering multiple bus routes . While the first definition is an extension from the common TTV definition used for private transport, the second definition is specially established for measuring PTTV of each bus service.

Day-to-day PTTV definition derived from private transport TTV

TTV_{d2d} is commonly calculated from the average travel time values of multiple days within a certain time window, or using the floating car travel time on the same study sites (Chien and Liu 2012; Oh and Chung 2006). This section extends this common definition of TTV to define PTTV, where the term "similar trips" means vehicle traversing on the same road section and within the same time period. We measure the variability of travel time using the Coefficient of Variation (CV) of travel time, the well-accepted measure of travel time variability in literature. CV is chosen as a meaningful comparison between two or more magnitude of variations. The TTV_{d2d} can be calculated as CV_p in equation (1).

$$CV_p = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,p} - \overline{TT}_p)^2}}{\overline{TT}_p} \qquad (1)$$

Here,

CV_p = CV of travel time (%) within time window *p* during *D* days,

$TT_{d,p}$ = mean travel time (s) of the vehicles traversing during time window *p* on day *d,*

$\overline{TT}_p$ = the average value of all $TT_{d,p}$ (s) within time window *p* during D days,

$$\overline{TT}_p = \frac{\sum_{d=1}^{D} TT_{d,p}}{D} \qquad (2)$$

The common definition of TTV_{d2d} can be extended to accommodate PTTV, in which PTTV is measured by the Equation (1). Each mean value $TT_{d,p}$ includes *all buses of all routes* passing the study corridor within a 30 minutes study time window on a working day. This definition of $PTTV_{d2d}$ is illustrated in Figure 2. Although individual vehicle travel time is available, the first definition of PTTV uses the mean travel time obtained from each day and period ($TT_{d,p}$) to measure only the day-to-day PTTV. The reason has been described in the "Travel time variability in literature" section of this paper. This paper terms this variability as ***day-to-day PTTV on corridor level*** ($PTTV_{d2d,c}$).

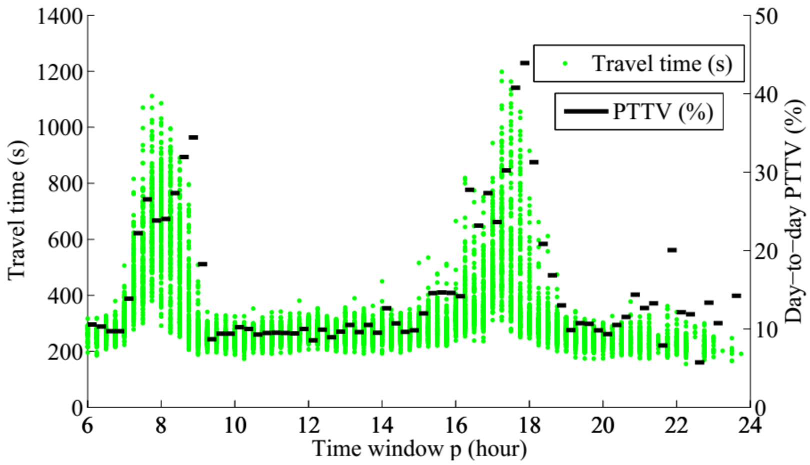


Figure 2 Observed $PTTV_{d2d,c}$ on Coronation Drive, Brisbane [INSERT]

$PTTV_{d2d,c}$ definition is useful for traffic managers in monitoring the day-to-day variability of bus travel time in general. Having the same method to calculate TTV enables effective comparison of the variability between different modes of transport, for instance between public and private transport.

Day-to-day PTTV definition using additional data of transit vehicles

Public transport often allows tracking of each individual vehicle on a specific service. This sub-section establishes another definition of *day-to-day PTTV* to take advantage of the additional information. The definition aims for monitoring transit performance and facilitating timetable adjustments. The term “similar trips” refers to the buses on the *same route* and *service*, because these buses are scheduled to travel time similarly.

$$CV_{r,s} = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,r,s} - \overline{TT}_{r,s})^2}}{\overline{TT}_{r,s}} \quad (3)$$

Here

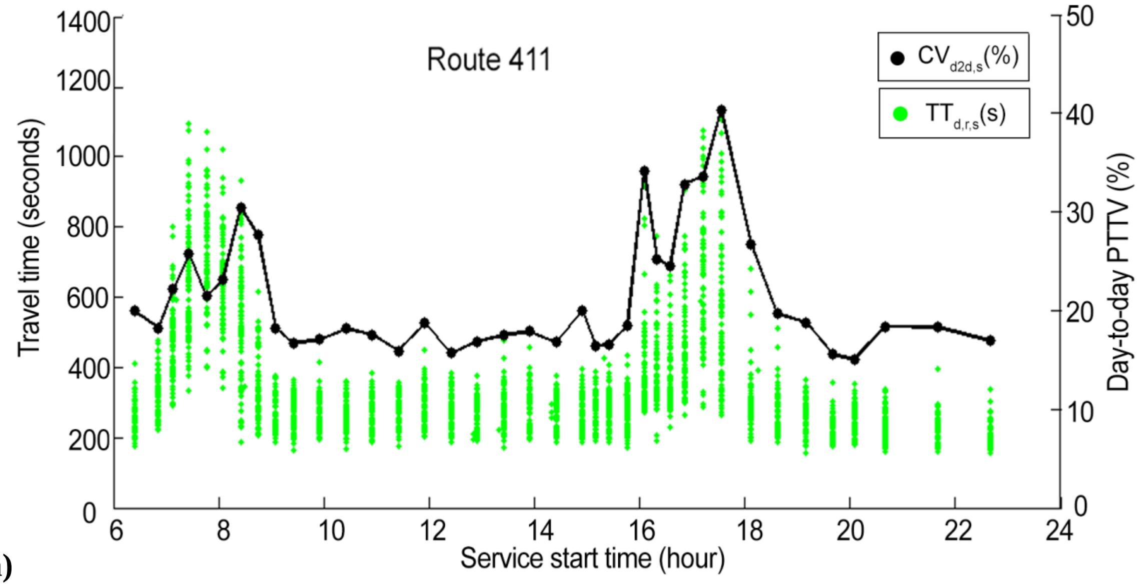
$CV_{r,s}$ = CV of travel time (%) of route *r* and service *s* during *D* days,

$TT_{d,r,s}$ = d^{th} individual travel time sample (s) of the bus of route *r* and service *s* on day *d,*

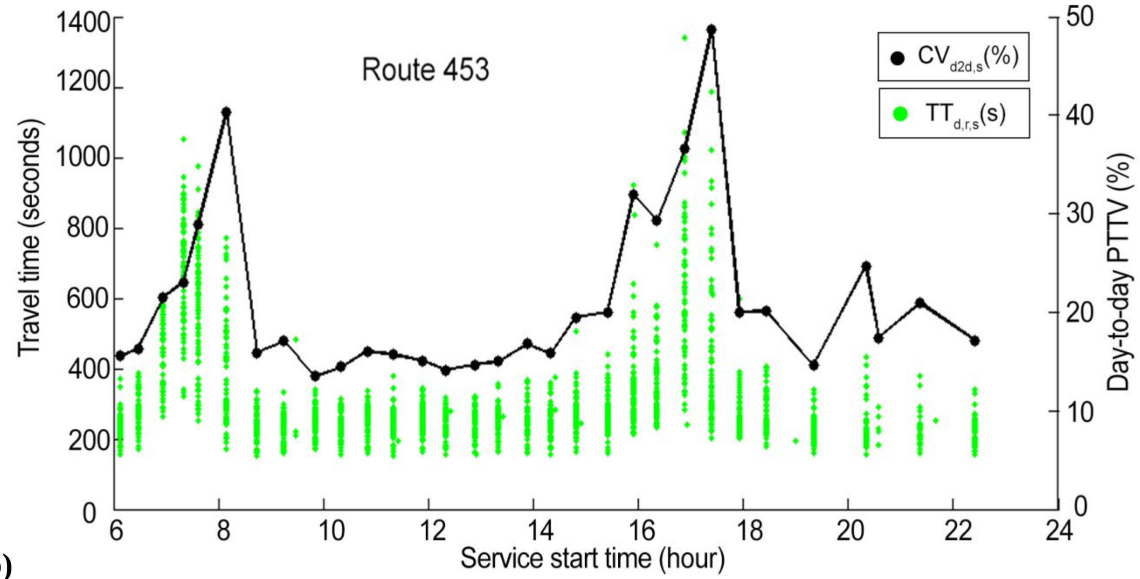
$\overline{TT_{r,s}}$ = the average value of all $TT_{d,r,s}$ (s) of route *r* and service *s* on all day,

$$\overline{TT}_{r,s} = \frac{\sum_{d=1}^{D} TT_{d,r,s}}{D} \quad (4)$$

This definition separates from the common measurement of private transport TTV by making use of the additional data of public transport. Each value of $TT_{d,r,s}$ includes only an individual bus of the specific service on a specific route. Figure 3 illustrates the definition using the four routes running along the Coronation Drive. It shows the day-to-day PTTV of services during off-peak periods are relatively low, indicating high reliability. The variability follows the same pattern as the congestion increases and reduces. Afternoon congestion shows a small peak of $CV_{r,s}$ before the main peak congestion at the school-off time when secondary school students are traveling home. This paper terms this variability as ***day-to-day PTTV on service level*** **(**PTTV$_{d2d,s}$).This second established definition of PTTV is useful for transit operators in scheduling, particularly in deciding the timetable and recovery time along with discovering the multiple day reliability performance of each service because it is defined by individual bus travel time.



(a)



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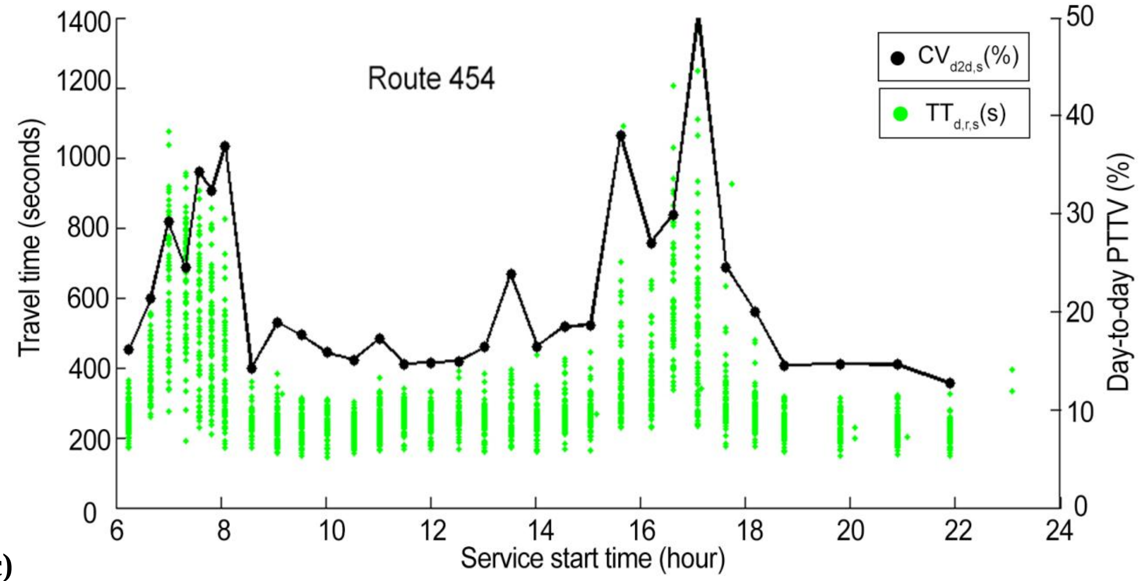
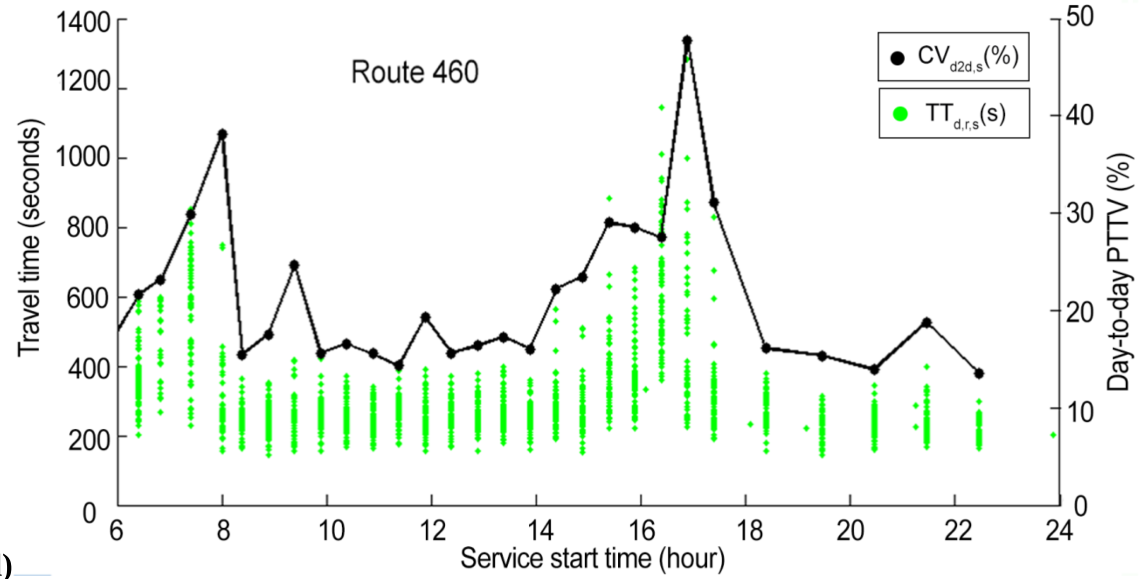


(c)



(d)

Figure 3 Observed $PTTV_{d2d,s}$ on Route: (a) 411, (b) 453, (c) 454 and (d) 460 [INSERT]

The aforementioned two definitions are further discussed as below:

Day-to-day PTTV on corridor level ($PTTV_{d2d,c}$) is the extension of the widely used definition of TTV to public transport. The definition reflects the PTTV in general by considering all passing buses, which enables meaningful comparison with other modes of transport. For instance, $PTTV_{d2d,c}$ provides insights on how the consistency and dependency of public transport modes are compared to private counterparts.

Day-to-day PTTV on service level ($PTTV_{d2d,s}$) measures TTV of a specified route service. The individual bus travel time samples on multiple days are used for TTV calculation. These individual buses are planned to travel similarly as they are on the same route and service. The variations in their travel times show the patterns of TTV and indicate service performance. Significantly, as it is a more focused scale compared to the first definition. $PTTV_{d2d,s}$ facilitates investigating the sources of unreliability and optimizing the timetables. The definition of $PTTV_{d2d,s}$ is more useful as it provides more information on individual vehicle performances, which can be used on more transit planning purposes.

DAY-TO-DAY PUBLIC TRANSPORT TRAVEL TIME DISTRIBUTION ANALYSIS

The previous section established definitions of PTTV and identified $PTTV_{d2d,s}$ *as* the most useful definition. This section analyzes the probability distribution of travel time to investigate *the nature and shape of* $PTTV_{d2d,s}$. For instance, a uniform distribution denotes no variability, while a long tail skewed distribution shows the bus could experience high and unreliable travel time. Travel time distribution is also essential in public transport planning. Resource allocations such as recovery time and timetable optimization are not often planned on the basis of average travel time, but on minimizing the opportunity that any journey would exceed the scheduled time (Moghaddam et al. 2011). However, the literature on public transport travel time distribution is still limited and inconsistent, exploring only common distributions at limited time-of-the-day, and revealing symmetric types of distribution (Taylor 1982), skewed distribution (Andersson et al. 1979) or both of them (Mazloumi et al. 2010) as the descriptor of public transport travel time.

For the analyses on $PTTV_{d2d,s}$ a comprehensive seven-step approach is applied to all services of Route 411 – the busiest bus route on the Coronation Dr. The analysis aims to test all types of probability distribution which neglects only the discrete types of distribution (e.g. Binominal, Negative binominal, Poisson) as well as Uniform and limited samples distributions (Triangular, Rectangular) because the nature of travel time is continuous. The list of 23 fitted distribution types includes: Beta, Birnbaum-Saunders, Burr, Chi-Squared, Dagum, Erlang, Error, Exponential, Frechet, Gamma, Generalized Pareto, Inverse Gaussian, Levy, Logistic, Log-logistic, Lognormal, Nakagami, Normal, Rayleigh, Rician, Pareto, t location-scale and Weibull.

Seven-step approach for public transport travel time distribution analysis

Travel time samples of each service are fitted by the Maximum Likelihood Estimation (MLE) method to estimate the parameters of each distribution. Most existing studies of travel time distribution analysis performed one of the three common goodness-of-fit tests named Chi-Squared; Kolmogorov-Smirnov (KS); and Anderson-Darling to find whether the data follows the specified distribution (hypothesis H0). Any p-value larger than the significance level (α) fails to reject H0 and the distribution is considered as significantly fitted with the data. However, this method has two key drawbacks (Durbin 1973). Chi-squared requires large sample size, while the original critical values of KS and Anderson-Darling tests are not valid if parameters are directly estimated from the data.

Literature offers other approaches which solve the aforementioned problems, but they also have their own disadvantages. First, the information creation technique such as Bayesian Information Creation (BIC) (Schwarz 1978) measures the relative quality of a statistical model by trading off the complexity (by considering the number of parameters) and goodness-of-fit of the fitted distribution (by considering the maximized value of the log-Likelihood). However, the BIC statistic is difficult to interpret. The fitted distribution with the lowest BIC is the "best" descriptor of the data, without a hypothesis testing to validate the goodness-of-fit. Second, the best fitted distribution could be examined graphically by using the probability plot, histogram, stem & leaf plots, scatter plot, or box & whisker plots. This graphical approach does not provide a reference point so that multiple distributions can be compared within multiple time periods. Third, recent goodness-of-fit tests such as Lilliefors test (Lilliefors 1967) extends the KS test by determining the critical value by a Monte Carlo simulation, which enables estimating the distribution parameters from the data. However, the critical values table supports only a few limited types of distributions, restricting the study to a few selected distributions.

To overcome the limitation of the existing approaches in travel time distribution analysis, this paper extends the Liliefors test to support all types of distribution. Instead of using any tables from Liliefors, KS or Anderson-Darling, we use parametric bootstrapping – a Monte Carlo simulation method (D'Agostino and Stephens 1986) for calculating KS critical value of each distribution type at each service. The parametric bootstrapping KS test identifies the list of distribution types that passed the KS, but does not provide a measure to compare the fitness of each distribution type if multiple types are accepted. A hybrid approach is then used, in which the top five distribution types in the number of passed KS tests are chosen as the five candidates for the descriptor of bus travel time. BIC statistic (Schwarz 1978) is then calculated to compare the goodness-of-fit of the five candidates where the one with lowest BIC has the best fitness to the bus travel time data. The descriptor of bus travel time will then pass the most number of KS test, while having the lowest BIC statistic value. The hybrid method could be described in 7 steps.

Step 1: Consider each type of distribution. MLE method is employed to estimate distribution parameter(s) from bus travel time data.

Step 2: Generates random data samples from the studied distribution using the parameter(s) from Step 1.

Step 3: Use MLE to re-estimate distribution parameter(s) from the generated data. The parameter(s) is used to build theoretical cumulative distribution function (c.d.f) *F(x)*

Step 4: Calculate the KS statistics D_N^*, i.e., maximum difference between the empirical distribution function (e.d.f.) $S_N(x)$ from the generated data (Step 2) and the theoretical c.d.f. *F(x)* (Step 3)

$$D_N^* = \max\left|S_N(x) - F(x)\right| \qquad (5)$$

Step 5: Repeat Step 2 to Step 4 a large number of time (say 10000) to gather the set of D_N^*. Since significance level (α) equals 0.05, the 95th percentile of the set is chosen as the critical value D_C.
Step 6: Compute the observed KS statistic D_N between the e.d.f. from the bus travel time data and the c.d.f. using the parameter(s) from Step 1, and compare it to the simulated critical value. If $D_N < D_C$, the test fails to reject the null hypothesis that the distribution could describe bus travel time data.
Step 7: BIC statistics are calculated for each candidate distribution from Step 6.
The BIC can be formulated as follows (Schwarz 1978)

$$BIC = k \ln n - 2 \ln L_{max} \tag{6}$$

Where:
n = number of observations
k = number of parameters to be estimated
L_{max} = maximized value of the likelihood function of the estimated distribution
This seven-step approach investigates the best descriptor of public transport travel time.

Analysis results and discussion

The Step 6 of the seven-step approach reveals five candidates of bus travel time distribution: Burr, Gamma, Lognormal, Normal and Weibull. While Normal and Lognormal are commonly used in public transport studies, the other three are relatively new in the area. The KS test results and histogram of each distribution type, along with the lowest 2 distribution types in BIC statistics are presented in Table 1. The following presents each aforementioned candidate to justify its overall goodness-of-fit to the bus travel time data.

Table 1 Descriptive statistic of travel time distribution analysis

Service	Observations	Skewness	Kurtorsis	KS test with bootstrap resampling (1 for Accepted, 0 for rejected)					Hatigan Dip test		Lowest BIC	2nd lowest BIC
				Burr	Normal	Log-normal	Wei-bull	Gam-ma	Dip stat	p-value		
6:51	103	1.2	7.05	0	1	1	0	1	0.04	0.38	Lognormal	Gamma
7:08	93	0.8	3.81	1	1	1	1	1	0.06	**0.02**	Lognormal	Gamma
7:26	104	0.4	2.68	1	0	1	0	1	0.05	0.06	Gamma	Lognormal
7:46	95	0.2	4.03	0	0	0	0	0	0.03	0.97	Normal	Burr
8:05	91	-0.6	3.84	1	1	1	1	0	0.03	0.77	Weibull	Normal
8:25	91	0.2	2.57	0	1	0	1	1	0.04	0.21	Weibull	Gamma
8:45	92	1.0	3.39	1	0	1	1	1	0.03	0.70	Lognormal	Burr
9:05	79	0.6	2.25	1	1	1	1	1	0.02	0.99	Lognormal	Gamma
9:25	120	0.0	1.94	0	1	1	1	1	0.05	**0.04**	Normal	Gamma
9:55	109	0.8	3.11	1	1	1	1	1	0.03	0.55	Burr	Lognormal
10:12	93	0.2	2.03	1	1	1	1	1	0.04	0.28	Gamma	Lognormal
10:55	109	0.5	2.56	1	0	1	1	1	0.04	0.14	Lognormal	Gamma
11:25	96	-0.2	2.18	0	0	0	0	0	0.04	0.44	Weibull	Normal
11:55	94	0.2	2.55	0	1	1	0	1	0.04	0.55	Gamma	Lognormal
12:25	108	0.0	2.23	0	0	0	0	0	0.03	0.82	Normal	Gamma
12:55	110	0.2	2.48	1	0	1	0	1	0.03	0.46	Gamma	Lognormal
13:25	113	0.1	2.66	0	0	0	0	0	0.04	0.31	Normal	Gamma
13:55	102	0.0	2.26	1	1	0	1	1	0.04	0.13	Normal	Gamma
14:25	99	-0.2	1.98	0	1	1	1	1	0.04	0.31	Weibull	Normal
14:55	97	1.4	8.40	1	0	1	0	1	0.05	0.06	Lognormal	Gamma
15:10	102	-0.1	2.50	0	1	0	1	0	0.02	0.97	Normal	Weibull
15:25	100	0.0	2.70	0	1	0	0	1	0.03	0.82	Normal	Gamma
15:46	90	-0.1	2.61	1	1	1	1	0	0.03	0.97	Normal	Weibull
16:05	105	2.1	9.68	1	1	1	1	1	0.02	0.96	Burr	Lognormal
16:20	97	1.1	4.99	1	1	1	1	1	0.04	0.38	Burr	Lognormal
16:35	88	1.1	3.88	1	1	1	1	1	0.04	0.49	Burr	Lognormal
16:51	89	1.8	7.76	1	0	1	1	1	0.04	0.39	Lognormal	Burr
17:13	91	1.0	4.62	0	0	1	0	1	0.03	0.71	Lognormal	Burr
17:33	101	1.0	3.21	0	0	1	1	1	0.03	0.80	Lognormal	Burr
18:07	72	2.2	9.54	1	1	1	1	1	0.03	0.86	Burr	Lognormal
18:37	93	1.3	6.30	1	0	1	0	1	0.03	0.61	Lognormal	Burr
19:10	77	0.2	2.21	0	0	1	0	1	0.04	0.51	Gamma	Lognormal
19:40	98	1.0	4.44	0	0	1	0	1	0.03	0.95	Lognormal	Gamma
20:05	106	0.2	2.56	0	0	1	0	1	0.03	0.78	Gamma	Lognormal
20:40	105	0.6	2.65	0	0	1	0	1	0.03	0.84	Lognormal	Gamma
21:40	85	1.2	5.53	0	1	1	1	1	0.02	0.99	Lognormal	Gamma
22:40	79	0.7	3.37	0	1	1	0	1	0.04	0.72	Lognormal	Gamma

The *Burr distribution* has been recently used in traffic engineering to model urban road travel time (Susilawati et al. 2011). Burr distribution is described as a heavy-tailed, highly-skewed distribution. Table 1 shows that while the Burr distribution only passed the KS test at 18/37 services, it is the best fitted distribution where bus travel time is high left skewed and long tailed, especially with a range of travel time with very high occurrences. However, this travel time pattern appears in only a few services.

The *Weibull distribution* has been widely used to represent travel time on arterial roads (Al-Deek and Emam 2006) and especially on duration-related studies such as traffic delay durations (Mannering et al. 1994) and waiting time at unsignalized intersections (Hamed et al. 1997). Weibull distribution has been described as flexible representing right-skew, left-skew and also symmetric data. The BIC results show that Weibull is almost always within the top 2 in negative skewed travel time patterns. As the services with negatively skewed distribution are limited in the dataset, Weibull distribution has the lowest BIC statistic value in only 3 services.

The *Normal distribution* has been suggested as the descriptor of bus travel time in a number of studies (Mazloumi et al. 2010; Taylor 1982). It has a symmetric shape and its characteristics are thoroughly studied in statistics, which facilitates theoretical research. Normal distribution is still a strong candidate as the descriptor of bus travel time in this study by passing the KS test in 20/37 services and having the lowest BIC statistics in 8 services, most of which are in mid-peak period.

The tests results indicate the *Gamma* and *Lognormal distributions* to be superior. The *Gamma distribution* has been long considered one of the first candidates for distribution of travel time. Polus (1979) believed that travel time on arterial road would "closely follow" a Gamma distribution, and for this reason Dandy and McBean (1984) suggested Gamma distribution as the descriptor for in-vehicle travel time. *Lognormal distribution* is extensively used to represent bus travel time (Andersson et al. 1979; Mazloumi et al. 2010) due to the flexibility and ability to accommodate skewed data.

While the Gamma distribution passes the KS test in 30/37 service, the Lognormal distribution passes in only one less services (29/37 services). Both of them are the optimal descriptors of bus travel time with moderate skewness and kurtosis (i.e. absolute value of skewness smaller than 1 and kurtosis smaller than 3). This type of travel time pattern is dominant in the dataset, which is why Gamma and Lognormal passed most KS tests.

Both Lognormal and Gamma distribution are capable of modeling both heavy and light tailed data, but the Lognormal is better in representing higher skewed and longer tailed data, as it came with the Burr distribution in the top 2 lowest BIC statistic in several services. The BIC statistics also indicate that Lognormal is the best fitted distribution in more services than any other distribution types (14/37 services).

Another advantage of the Lognormal distribution is its mathematical characteristics that facilitate TTV studies. Lognormal distribution allows direct calculation of CV from its parameter.

$$\mathrm{CV} = \sqrt{e^{\sigma^2} - 1} \qquad (7)$$

The $\left(p \times 100\right)^{\text{th}}$ percentile θ, commonly used in many variability and reliability indicators, can be computed using the lognormal quartile function as in Equation (8)

$$\theta = F_X^{-1}\left(p\right) = e^{\mu - \sqrt{2} erfcinv(2p)\sigma},\ 0 \le p \le 1 \qquad (8)$$

where $erfcinv(x)$ is the inverse complementary error function. While there is no known closed form expression, the value of $erfcinv(x)$ can be approximated to the method described in Blair et al. (1976). Equation (8) also denotes that if the data is Lognormally distributed, the Lognormal parameters μ and σ can be easily estimated from the value of two percentile values ($p_1 \times 100$)-th percentile θ_1, and the ($p_2 \times 100$)-th percentile θ_2, which means the following equations can be obtained.

$$\begin{cases} p_1 = \frac{1}{2} erfc\left(-\frac{\ln(\theta_1) - \mu}{\sqrt{2}\sigma} \right) \\ p_2 = \frac{1}{2} erfc\left(-\frac{\ln(\theta_2) - \mu}{\sqrt{2}\sigma} \right) \end{cases} \quad (9)$$

The parameters of Lognormal can be calculated by solving Equation (10)

$$\begin{cases} \sigma = \dfrac{\ln(\theta_2) - \ln(\theta_1)}{\sqrt{2}\left[erfcinv(2p_1) - erfcinv(2p_2)\right]} \\ \mu = \ln(\theta_1) + \sqrt{2} erfcinv(2p_1)\sigma \end{cases} \quad (10)$$

Overall, Lognormal distribution provides excellent representation of the public transport travel time. It is recommended as the descriptor of public transport travel time variation due to its high performance and the attractive mathematical characteristics that facilitate TTV studies.

Hartigan Dip test for examining the bimodality

The histograms on Table 1 show some signs of bimodality on two services before and after the morning peak period. Testing the bimodality is best conducted with the Hartigan Dip test. Dip statistics express the largest difference between the empirical distribution function and a unimodal distribution function that minimizes that maximum gap (Hartigan and Hartigan 1985). If the *p-value* of the test is more than the significance value (chosen as 0.05), the data is concluded as unimodal distributed.

The results from Table 1 show that although the bimodality is significant in only two services, the distributions of travel time in many services before and after the morning peak period are also nearly bimodal (*p-value* slightly larger than 0.05). The bimodality of travel time is mainly caused by a mixture of congested and uncongested population of traffic. Earliness or excessive congestion on some days, or generally the spread of congestions could be the main reason. These services are within the congestion build-up and dissipation periods, where speed could be free flow or congested depends on a day-to-day basis. The study was conducted on inbound traffic only, which means the pattern is not repeated for the afternoon.

PROBABILISTIC APPROACH USING LOGNORMAL DISTRIBUTION FOR INDICATING PUBLIC TRANSPORT TRAVEL TIME VARIABILITY

Lognormal has been recommended as the descriptor of day-to-day public transport travel time in this study. This section investigates the use of *Lognormal distribution* to empirically indicate *day-to-day PTTV on service level* using a probabilistic approach.

TTV or travel time reliability is often indicated by one of the four measures (van Lint et al. 2008): statistical range, buffer time, tardy-trips or probabilistic approach. The probabilistic approach is one of the direct measures to evaluate travel time reliability. Bell and Cassir (2000) defined reliability as "the probability that [a] system can perform its desired function to an acceptable level of performance for some given period of time". The probabilistic approach measures the probability that travel time would be higher than a predetermined threshold under normal traffic conditions subject to day-to-day traffic flow fluctuations. The predetermined threshold is often defined as the median of travel time plus a certain amount of time, or a certain percentage of the median of travel time (van Lint et al. 2008). This section aims to use the p.d.f. of the Lognormal distribution to calculate the probabilistic indicator of PTTV. The probability that bus travel time is larger than a certain value from the median travel time is expressed by Formula (11).

$$\Pr\left(TT_{d,r,s} \geq A\right) \tag{11}$$

Where,

A = predetermined travel time threshold to be studied, e.g. A = $\alpha \times T50_{r,s}$ or A = $\beta + T50_{r,s}$

α = threshold multiplied with the median (e.g. 1.2)

β = threshold added to the median (e.g. 10 minutes)

$TT_{d,r,s}$ = travel time of the bus of route *r* which is scheduled to start at service *s* of day *d*

$T50_{r,s}$ = median value of the set of travel time samples of route *r* and service *s*

The p.d.f. of Lognormal distribution has the form as in Equation (12).

$$f_X\left(x\right) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left[\frac{\ln(x)-\mu}{\sqrt{2}\sigma}\right]^2}, \quad x > 0 \tag{12}$$

Where μ and σ are the two parameters of the Lognormal distribution. Mathematically, the probability $\Pr\left(TT_{d,r,s} \geq \alpha \times T50_{r,s}\right)$ or $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ is the integral of the p.d.f. $f_X\left(x\right)$ between threshold value $A = \alpha \times T50_{r,s}$ or $A = \beta + T50_{r,s}$ and the infinity.

$$P = \int_A^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} e^{-\left[\frac{\ln(x)-\mu}{\sqrt{2}\sigma}\right]^2} dx \tag{13}$$

Substitute $y = \ln\left(x\right)$, which means $x = e^y, dy = \frac{dx}{x}$. Equation (13) becomes.

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{lnA}^{\infty} e^{-\left[\frac{y-\mu}{\sqrt{2}\sigma}\right]^2} dy \qquad (14)$$

The problem in Equation (14) can be re-written into an equation of the complementary error function.

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{lnA}^{\infty} e^{-\left[\frac{y-\mu}{\sqrt{2}\sigma}\right]^2} dy = \frac{1}{2}\left[erfc\left(\frac{lnA-\mu}{\sigma\sqrt{2}}\right)\right] \quad (15)$$

Where *erfc*(x) is the complementary error function

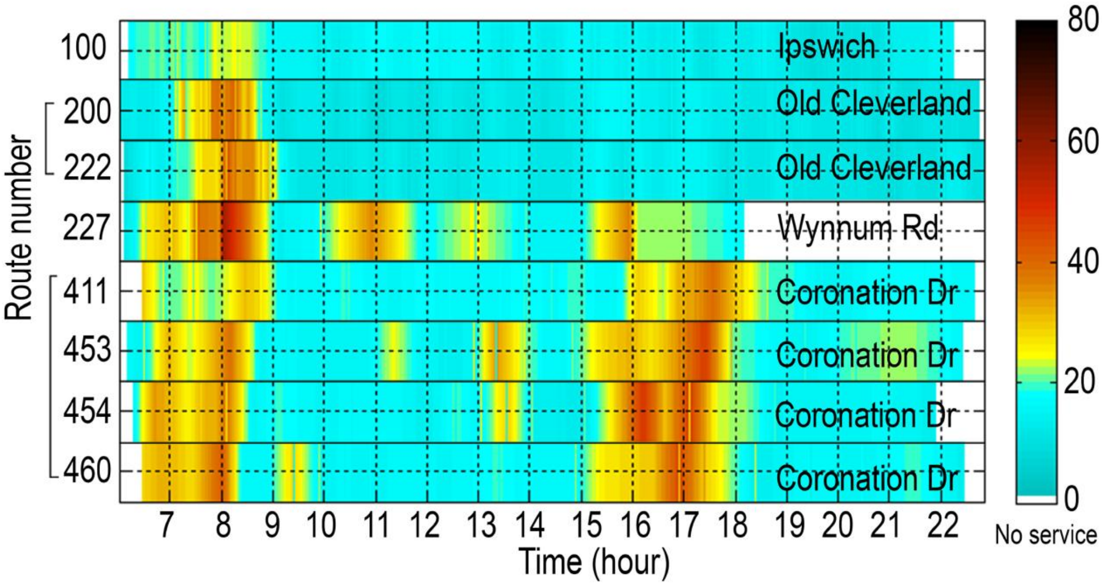
$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \qquad (16)$$

The value of *erfc(x)* can be rationally approximated (Cody 1969) to get the desired quantity. Equation (12)-(16) show that using the p.d.f. of the Lognormal distribution, the probability that the bus travel time exceeds a certain threshold from the median is found.

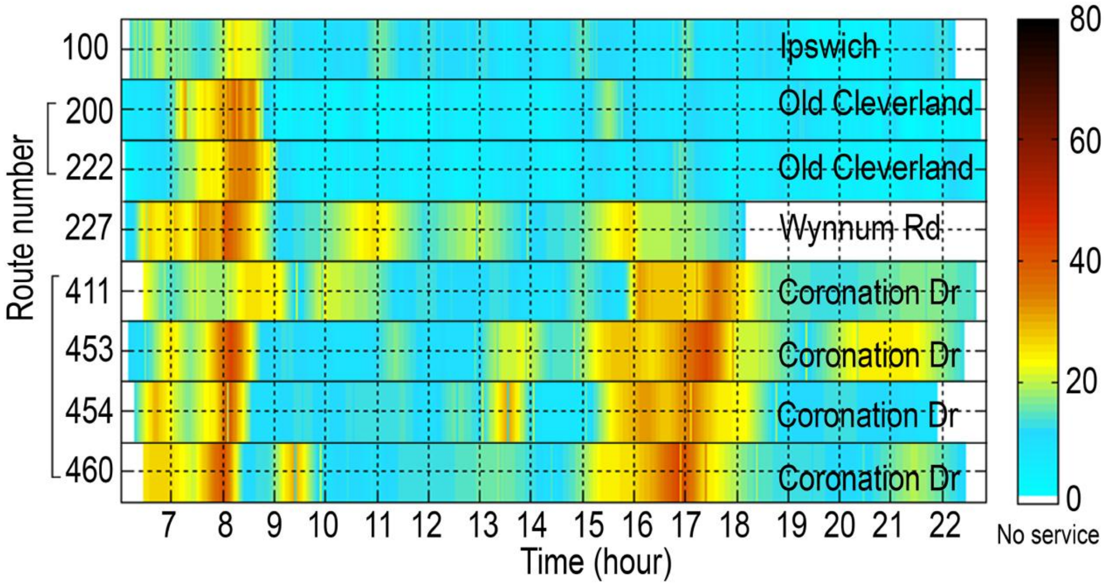
Public transport travel time variability map of some main routes in Brisbane

This sub-section uses the probability indicator in Equation (11) to show the PTTV at the 4 sites illustrated in Figure 1. The objective is to validate the applicability of the study in monitoring PTTV of multiple routes and corridors. Lognormal distribution is fitted to each set of data using MLE method to find the parameters σ and μ.

Figure 4(a) and Figure 4(b) show the PTTV maps of 8 bus routes along the 4 study sites. While Figure 4(a) demonstrates PTTV in terms of CV of travel time, Figure 4(b) demonstrates PTTV in terms of the probability that the travel time is higher than 20% of the median: $\Pr\left(TT_{d,r,s} \geq 1.2 \times T50_{r,s}\right)$. The 20% is chosen to be consistent with the threshold used by Van Lint *et al.* (2008), but any threshold can be used to calculate the probabilistic indicator. The two figures confirm that the proposed probabilistic approach captures the variability patterns on each site and indicates PTTV, and show very similar results to the popular approach using CV of travel time. Corronation Drive's routes travel times are highly varied during both morning and afternoon peaks as the corridor is directly connected with the Brisbane CBD. The routes from other corridors are only unreliable during morning peak periods. This section validates that the study can be applied to multiple routes over multiple sites to indicate PTTV.



(a)



(b)

Figure 4 PTTV map using: (a) CV of travel time and (b) $\mathbf{Pr}\left(\boldsymbol{TT}_{d,r,s} \geq 1.2 \times \boldsymbol{T}50_{r,s}\right)$ [INSERT]

A DISCUSSION ON PRACTICAL APPLICATIONS

The previous section confirms that the proposed probabilistic approach captures the PTTV, similar to the traditional CV approach. While CV is only useful for monitoring the PTTV, the proposed probabilistic approach can evaluate the probability of bus travel time over any predefined threshold.

First, the proposed method facilitates timetabling, especially in determining the recovery time. Taking the median value $T50_{r,s}$ from all historical travel time between two time points as the expected running time, transit operator would be interested in determining a recovery time value β added to $T50_{r,s}$ to accommodate the variance of travel time. This is equal to minimizing the probability that the observed travel time would be higher than the total scheduled travel time.

Minimize $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ (17)

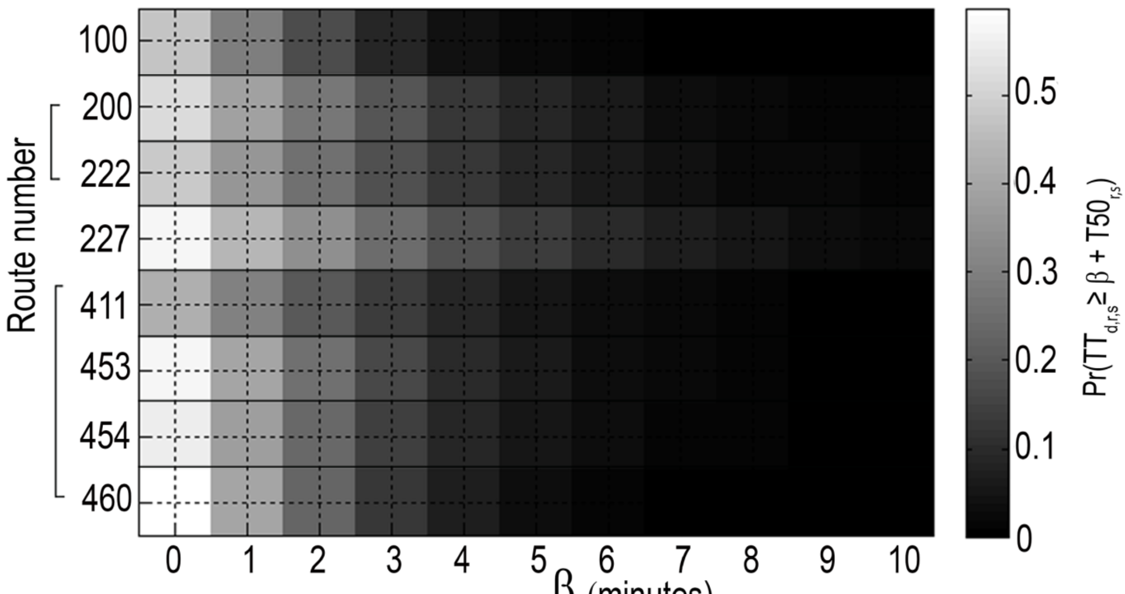
Where:

$T50_{r,s}$ = median value of the set of travel time samples of route *r* and service *s*, set as the expected running time

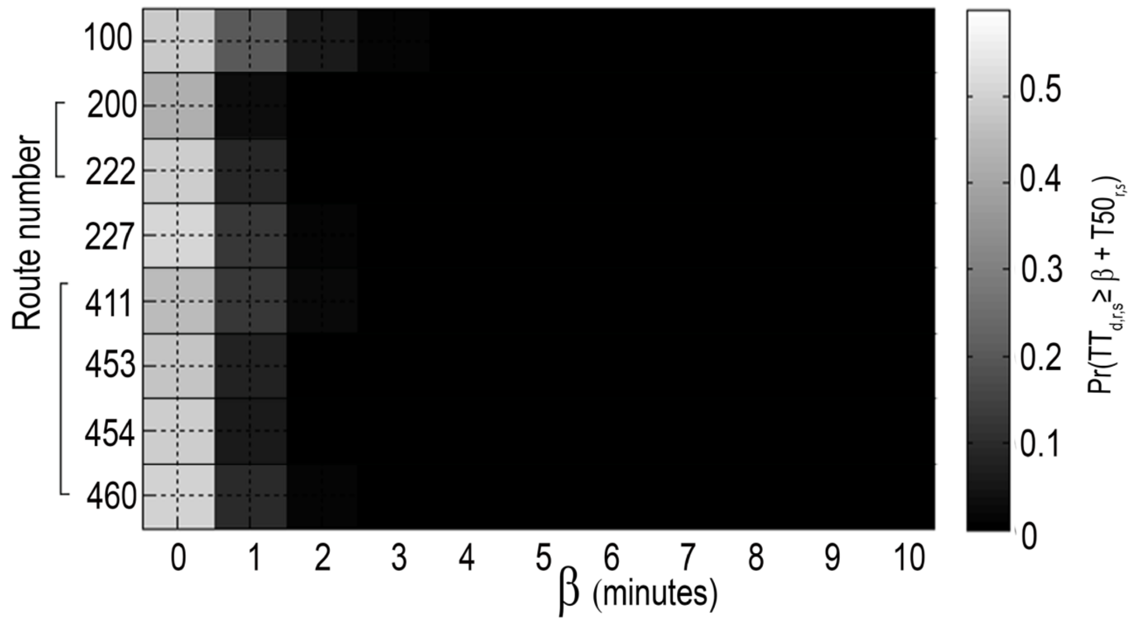
β = recovery time

$\beta + T50_{r,s}$ = total scheduled travel time

Figure 5 illustrates the value of $\Pr\left(TT_{d,r,s} \geq \beta + T50_{r,s}\right)$ when β varies from 0 to 10 for the different study routes where Figure 5 (a) is for 8:00 am and Figure 5 (b) is for 12:00pm. The figure clearly indicates that recovery time is dynamic over both route and time. A static constant recovery time for all the routes may not be optimal. For the study site, if transit operators aims for 90% of buses for on-time, then recovery time for morning period (8:00 am, Figure 5(a)) should be around 3 to 7 minutes depending on the route. For instance, 7 minutes for route 411 and 3 minutes for route 100. Similarly, for afternoon non-peak period it should be around 1 to 2 minutes. While most transit operators currently set a fixed scheduled travel time for all time-of-the-day, the information in Figure 5 facilitates a better timetabling to serve all passengers on-time. While adding more recovery time would also reduce commercial speeds, the proposed method enables analytical calculation to balance between high commercial speed and reliable travel time.



(a)



(b)

Figure 5 Value of $\Pr\left(\boldsymbol{TT}_{d,r,s} \geq \boldsymbol{\beta} + \boldsymbol{T}50_{r,s}\right)$ with varied $\boldsymbol{\beta}$ at: (a) 8:00 and (b) 12:00 [INSERT]

Travel time statistical studies also require the knowledge of travel time distribution. For instance, dynamic and stochastic traffic assignment usually assumes travel time as random variable and models travel time as a stochastic process follows a probability density function (Mirchandani and Soroush 1987). The distribution also shows the probability of excessive travel time (incidents), which is importance in route choice modeling (Watling 2006). The method for exploring travel time probability distribution in this paper facilitates these studies.

CONCLUSION

This paper established the public transport-oriented definitions of day-to-day TTV and analyzed its statistical characteristic. The first, corridor-level PTTV definition is an expansion of commonly used definition of TTV to include all buses that flowing through a corridor to provide the information of variability of buses in general. This is useful to compare between multiple modes of transport. The second, service-level PTTV definition includes only a specific bus route service, which can be used for performance measurement, and optimizing recovery time. The second definition on service level is the most useful as it enables service monitoring and recovery time planning.

The investigation of public transport travel time probability distribution introduced the comprehensive seven-step approach which allows fitting most of continuous probability distributions to all services. Each type of distribution is tested by both KS test with parametric bootstrapping and BIC method, identifying Lognormal distribution as the descriptor of day-to-day public transport travel time.

Using the Lognormal distribution p.d.f. to calculate probabilistic indicators of PTTV is useful in PTTV monitoring and recovery time optimization. In fact, data from 8 bus routes along 4 corridors in Brisbane confirmed the applicability of the proposed probabilistic method for PTTV indicators.

The definitions and modeling methods presented in this paper established a strong basis for future researches. Statistical analysis, especially the ones using the p.d.f. of Lognormal distribution, can be further investigated. The factors that causing the long tail of the public transport travel time distribution or high travel time variability can also be explored in future studies. This paper only uses TSP data, which does not provide dwell time at stops. For future work advanced data sources such as AVL data should be used to investigate all sources of travel time variability. The possible variables that contribute to the PTTV are dwell time, road congestion and any online tactics (public transport priority systems or bus bunching prevention strategies).

In the meantime, the findings of this paper are best suited for PTTV monitoring, recovery time optimization and statistical analysis of public transport travel time.

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